

**Multiplication**

**Key Language:** Double, twice as many, groups, the same, equal to

**Nursery**

•

**Concrete**

**Pictorial**

**Abstract**

**Conceptual Vairation:** double five.

## Multiplication

**Key Language:** Double, twice as many, groups, the same, equal to

### Reception

**ELG**

- Automatically recall (without reference to rhymes, counting or other aides) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally

**Resources**

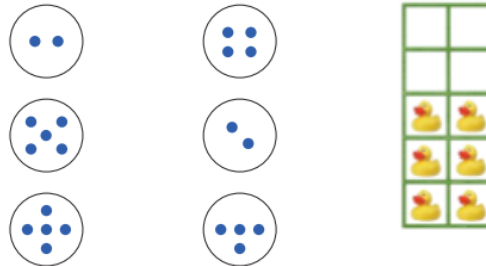
- Objects for counting
- Numicon
- Tens frames
- Multilink/unifix
- Part whole models
- Calculation frames

#### Concrete

#### Pictorial

#### Abstract

**Understanding equal groups**



Part whole models/tens frames to complete number sentences.

4 split into equal groups can be

1 and 1 and 1 and 1 or 2 and 2.

**Understanding doubling 'twice as many'**

Use physical objects to show doubling as two equal groups or twice as many

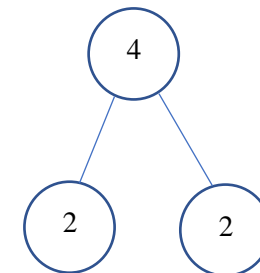


**Doubling ladybirds**

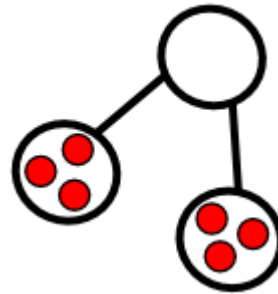
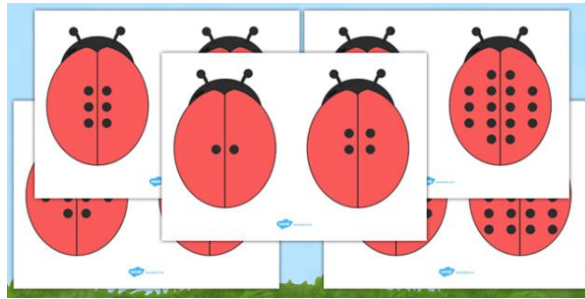
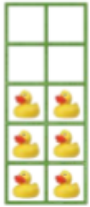
Children can see two groups being made when they paint one side and press together.

Use part whole models to represent two equal groups.

Use part whole models with numbers instead of objects or dots.



c

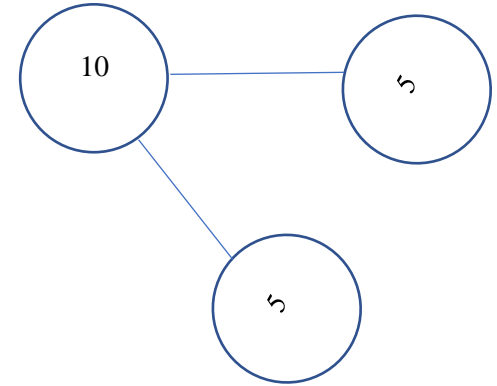
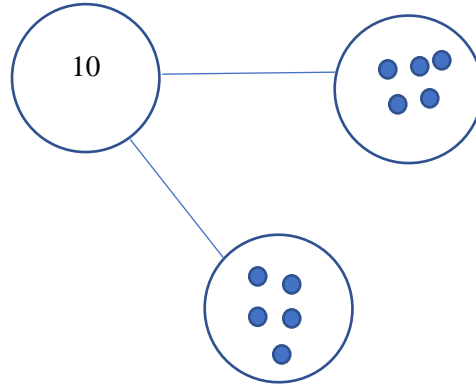
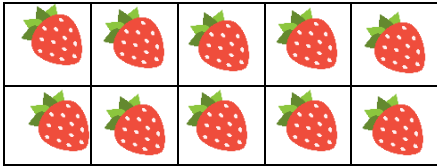
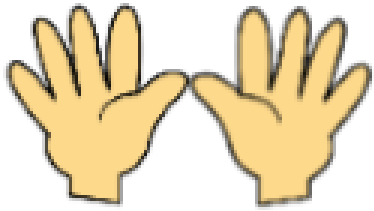


Use number sentences:

2 doubled is 4

$$2 + 2 = 4$$

**Conceptual Vairation:** double five.



# Multiplication

**Key Language:** Group, equal, double, twice as many, rows/arrays, equal to, product

## Year 1

### National Curriculum

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

### Resources

- counters/objects for counting
- part whole models
- multilink
- 

### Concrete

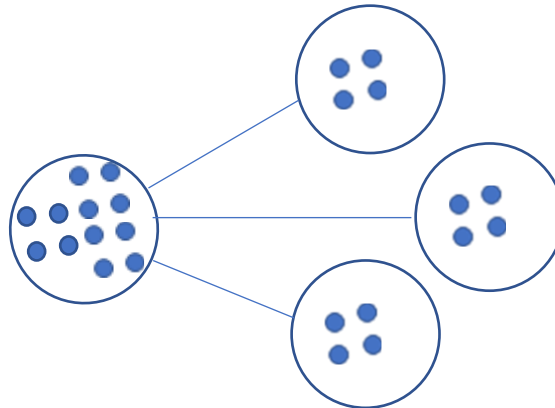
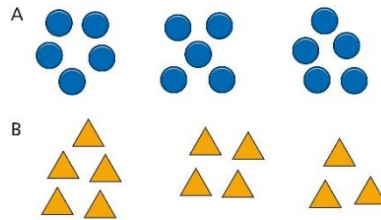
#### Making equal groups



Children to understand the meaning of equal groups. Each group must have the same amount in each.

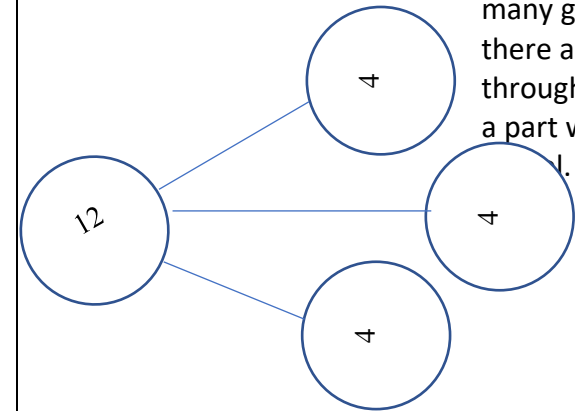
### Pictorial

Children to organise groups to ensure that they are equal



### Abstract

children to represent the equal parts and how many groups there are through using a part whole model.



Children to use sentences to represent the equal groups.

There are 3 equal groups of 4

**Adding equal groups (2,5,10)**

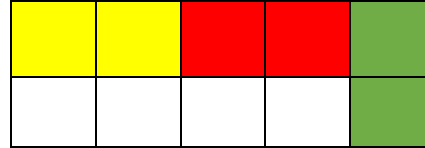
Pupils start to count in multiples of 2 and multiples of 10, then progress to counting in multiples of 2, 5 and 10 supported by discrete, countable representations.



4      6

2

Use tens frames to support children in counting in equal groups.



Children to use number tracks to support them in counting up in equal groups/steps.

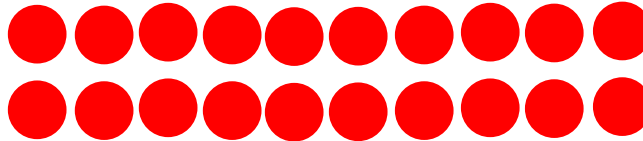


**Making equal rows**



There are 10 toy soldiers in one row.  
2 tens = 20  
There are 20 toy soldiers altogether.

Use arrays as a way of representing groups and how many within a group.



There are 2 groups of 10.  
2 tens are 20

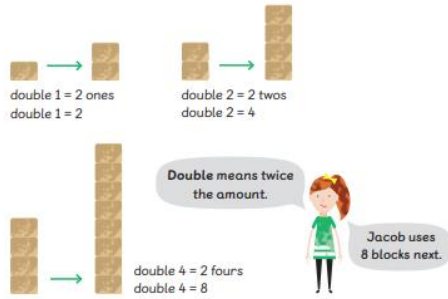
Children can make the link between equal groups and repeated addition.

Use number sentences from this.

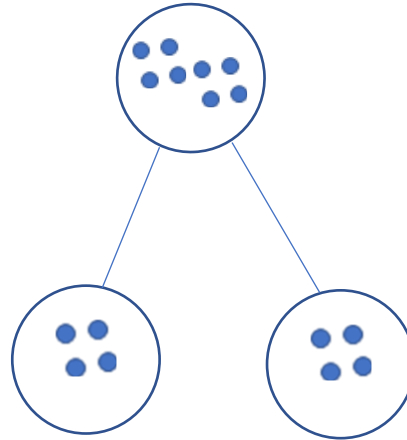
$$10 + 10 = 20$$

### Making doubles

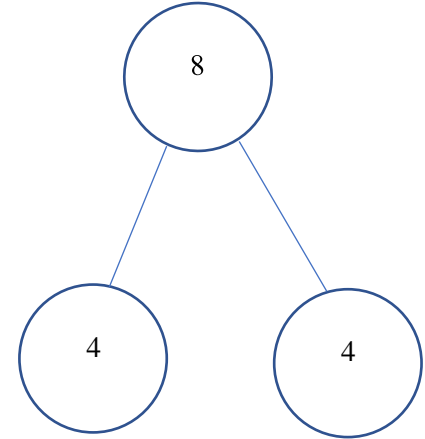
Start with using objects and getting children to show what doubling means.



Use a part whole models to represent two equal groups coming together to make the whole.



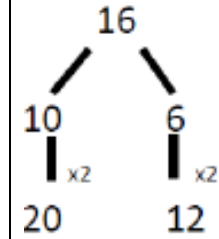
use part whole models to represent two equal groups coming together to create the whole.



Or use stem sentences- children should understand that double means 2 groups of the same number

4 doubled is 8  
2 groups of 4 make 8

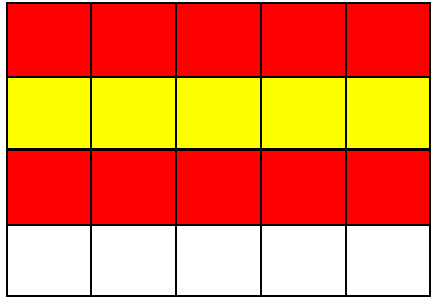
Partitioning to double then adding together



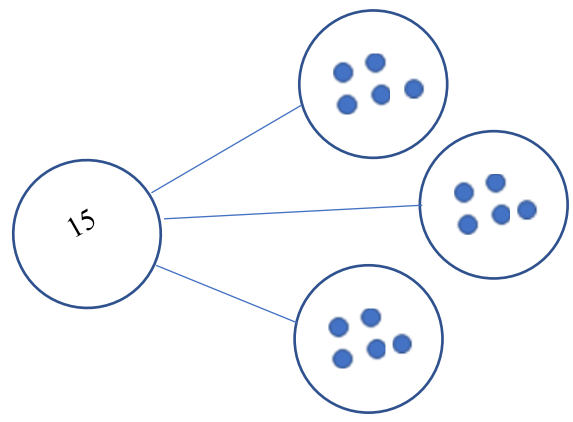
**Conceptual Variation: How many oranges are there all together?**



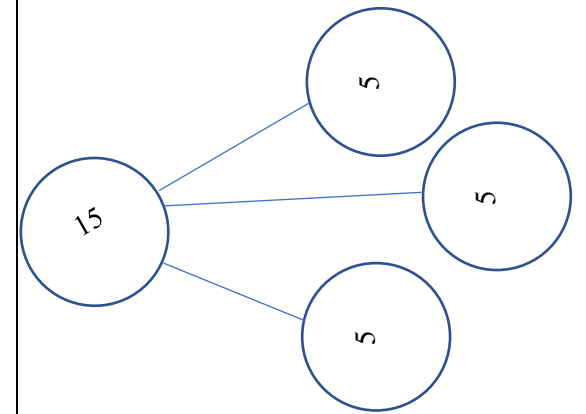
3  
of 5



groups



Or count in steps of 5



Or children should use stem sentences and their understanding of number bonds

		5	10	15		3 groups of 5 3 fives are 15
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## Multiplication

**Key Language:** Double, times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as', array, commutative

### Year 2

#### National Curriculum

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

#### Resources

- Counters
- Number tracks
- Tens frames
- Part whole models
- Multilink
- Number lines
- Dot cards

#### Concrete

##### Multiplication as equal groups

Recognise that the groups are equal.

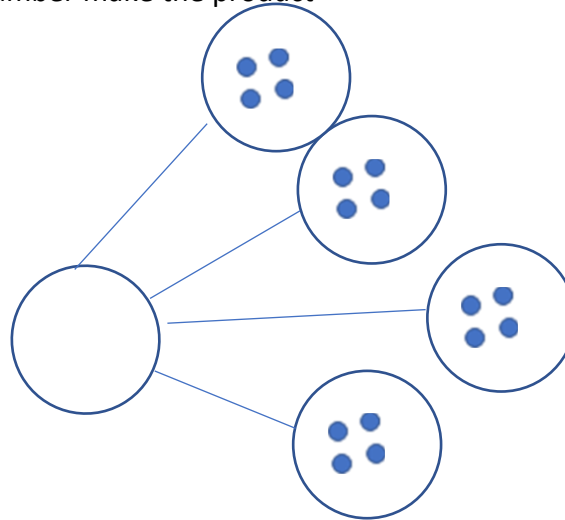
4 groups of 3

4 3's



#### Pictorial

Part whole models to show how many groups of a number make the product



Arrange

counters into equal groups and count in steps of how many are in each group.

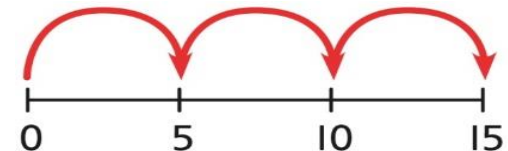
#### Abstract

Multiplication as repeated addition. Children to understand that they are adding on the same amount each time and can therefore use number lines and addition sentences to support them in finding the product.

$$4 + 4 + 4 =$$

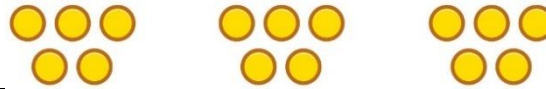
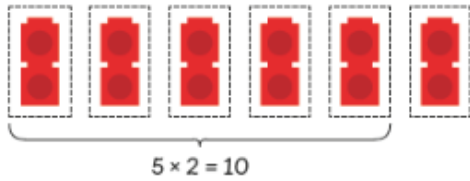
3 groups of 4

$$3 \times 4 =$$

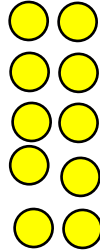


### Counting in 2's

Understand that counting in two's means counting 2 in each group



Children to use arrays to support them in counting in two's- represent the equation using counters.



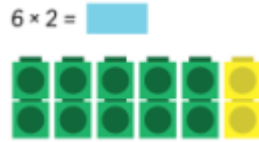
Children to use number tracks to support them in counting in twos.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

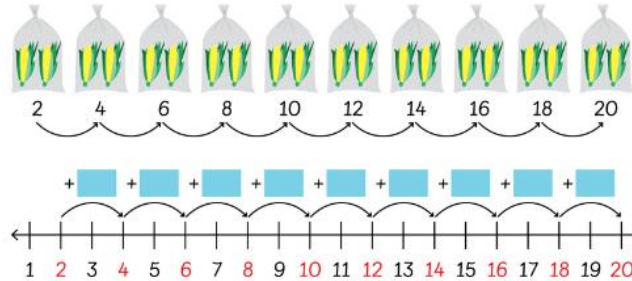
Children to use recall of two times tables to solve equations.  
 $5 \times 2 = 10$

**Associated facts**

To use key facts to support retrieval of other key facts- key facts to include 10s 5s and 2s



number lines used to represent adding on \_\_\_ more groups.



Abstracting mentally-

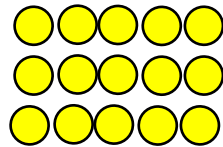
I know  $5 \times 2 = 10$  so  $6 \times 2$  is 2 more than 10.

**Counting in 5's**

Understanding that counting in fives means there are five in each group and increase by 5.



Use arrays to show grouping in fives.



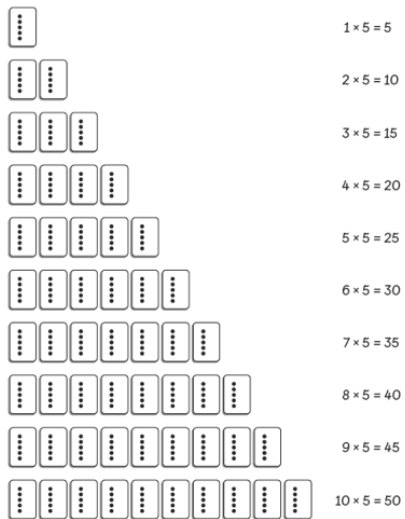
Use repeated addition

$$5 + 5 + 5 =$$
$$3 \times 5 =$$

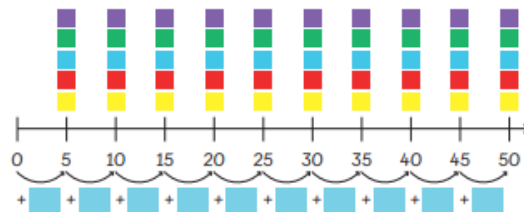
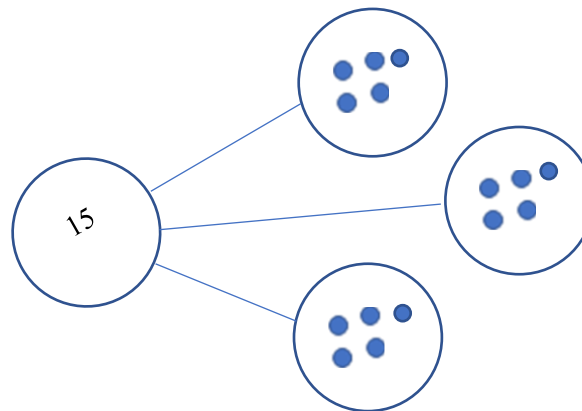
Children to use known number facts to support them in solving the equation:

$$2 \times 3 = 10 + 5 = 15$$

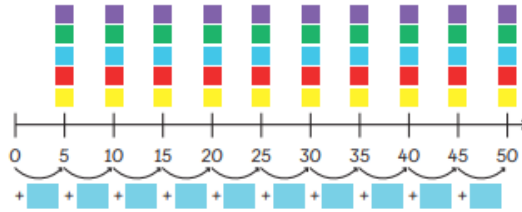
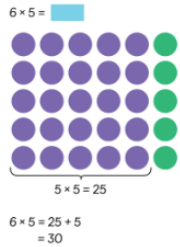
Use dot cards to show that numbers are increasing by 5 as you work further up the five times tables



Part whole models



**Associated facts**



Abstracting mentally-

I know  $5 \times 5 = 25$  so  $6 \times 5$  is 5 more than 25.

**Counting in 10's**



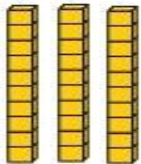
Count in steps of ten.  
10, 20, 30



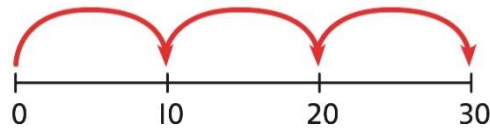
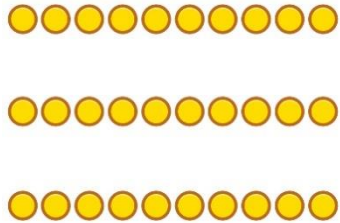
Children understand the tens column increases by 1 each time 10 is added.



Use dienes to represent 1 group of 10



Use related facts of counting in tens to support children in recalling times table facts  
children to use arrays and number lines to support them in understanding that when you are multiplying, all groups are equal.



Use repeated addition to show the same amount being added on.

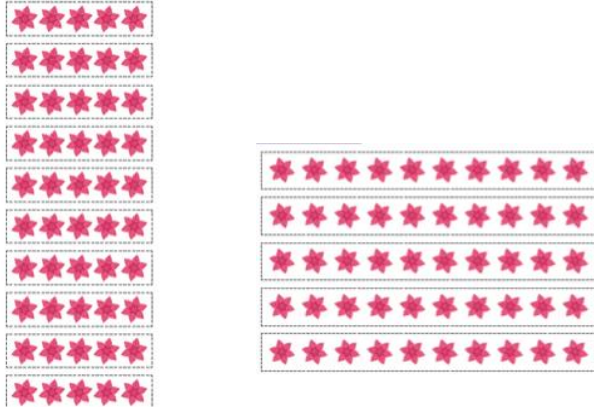
$$10 + 10 + 10 = 30$$

3 groups of 10  
 $3 \times 10$

Children to use known facts from the ten times tables to support them in quick recall.

$$4 \times 10 = 40$$
$$3 \times 10 = 40 - 10$$
$$3 \times 10 = 30$$

**Understanding commutativity**

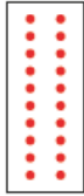


Children to use arrays to represent two equations with the same numbers in but swapped. E.G

10 groups of five or  $10 \times 5$   
5 groups of ten or  $5 \times 10$

Children to explore what this shows

continue to represent arrays using pictures to show that multiplication is commutative.  
Children to use repeated addition to show the difference in grouping.



$10 \times 2 = 20$   
 $2 + 2 + 2 +$   
 $2 + 2 + 2 +$   
 $2 + 2 + 2 +$   
 $2 = 20$



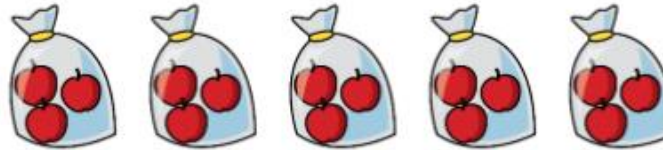
$2 \times 10 = 20$   
 $10 + 10 = 20$

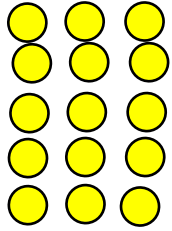
Children to use repeated addition and multiplication equations without concrete resources.

$5 \times 2 = 10$   
 $2 + 2 + 2 + 2 + 2 = 10$

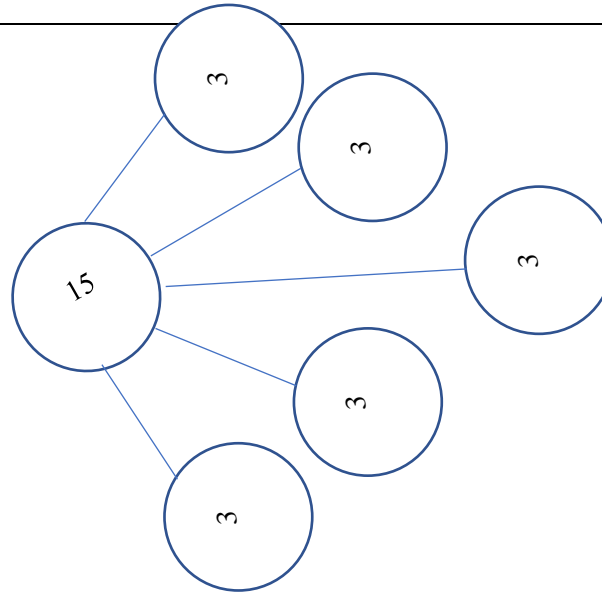
$2 \times 5 = 10$   
 $5 + 5 = 10$

**Conceptual Variation:**





$$3 + 3 + 3 + 3 + 3 = 15$$
$$5 + 5 + 5 = 15$$



$$5 \times 3 = 15$$

Or

$$3 \times 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 + 5 + 5 = 15$$

## Multiplication

**Key Language:** Double, times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as', array, commutative

### Year 3

#### National Curriculum

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

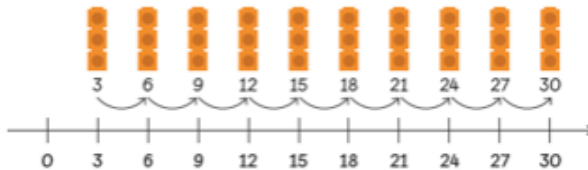
#### Resources

- counters
- Dienes
- Place value grids
- Number lines/tracks
- Multilink
- Part whole models
- Bar models
- Place value counters

#### Concrete

##### Multiplying by 3

Count in steps of 3 using a number line. Children to understand that at each jump we add on three more than the previous number. (building additive understanding of multiplication-supporting from year 2 )



#### Pictorial

Use arrays to support the children in representing a multiplication equation.



18					
3	3	3	3	3	3

#### Abstract

Children to use known number facts to support them in solving other multiplication equations



$$2 \times 3 = 6$$

So

$$4 \times 3 = 12$$

The amount of groups has doubled so the product will double.

$$4 \times 3 = 12$$

So

$$5 \times 3 = 15$$

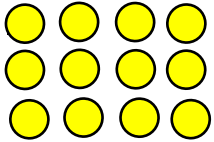
There is an additional group so the product is three more than the previous product.

$$6 \times 3 = 18$$

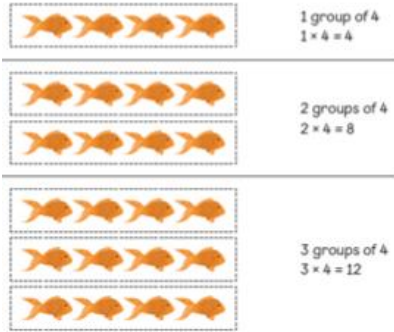
So

$$5 \times 3 = 18 - 3$$

**Multiplying by 4**



Children to use counters to create arrays to support them



12		
4	4	4

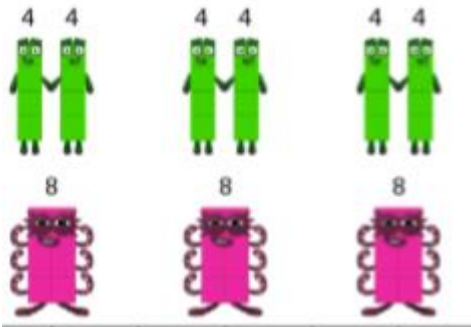
Children to use known number facts to support them in finding the products of multiplication equations.

Children should also begin to use quick recall if the four times tables.



$10 \times 4 = 40$   
So  $9 \times 4 = 40 - 4 = 36$

**Relationship between 4 and 8**



Children

to spend time understanding the relationship between 4 and 8. Understanding that 8 is double four and four is half of eight.

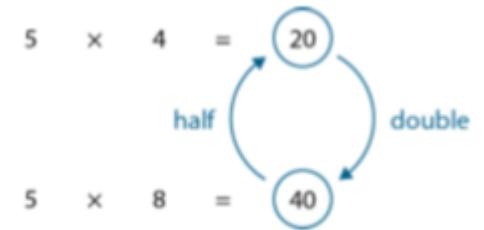
Use a number line to support understanding of the relationship between counting in fours and counting in eights.

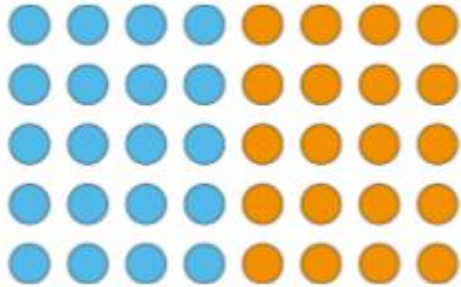
Children will see that you will not find all multiples of four in the eight times tables.



children to use bar models to abstract the relationship between four and eight times tables.

24					
4	4	4	4	4	4
8		8		8	



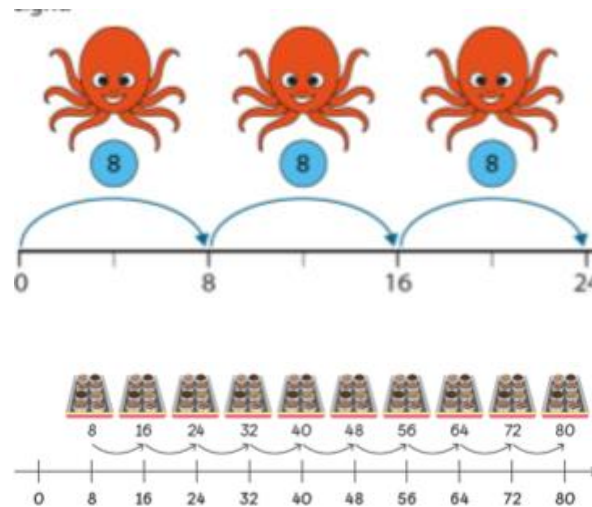


Create arrays ensuring that two parts of 4 are clear. Children to see that two groups of four are the same as 1 group of 8.

**Multiplying by 8**

Use arrays as they would have done previously to represent the multiplication equation. Ensure that children understand the concept of commutativity so that the array can be either way around.

Children to use a number line to show that they are adding on 8 each time they make a jump from one product to the next. Children should recognise that each jump represents one group of 8.



Children to model their understanding of how to use the bar model to support them in counting in eights from their understanding of counting in fours.

e.g  $4 \times 4 = 16$   
 $4 \times 8 = 32$

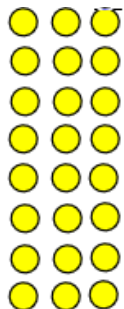
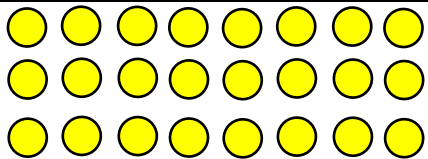
Children recall 8 times tables mentally without the need for counting on fingers etc. Children to recall 8 times tables using known number facts and 4 times tables to support.

E.g  $5 \times 8 = 40$   
 so  $6 \times 8 = 48$

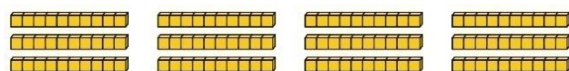
or

$6 \times 4 = 24$   
 So  
 $6 \times 8 = 48$

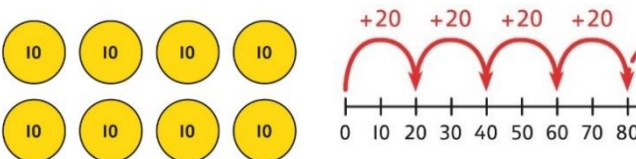
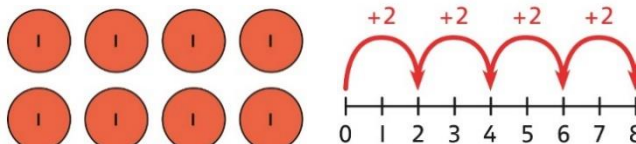
The amount in each group has doubled therefore the product will double.



**Using known facts to multiply multiples of ten**  
 using dienes explore what is the same and what is different about the two sets of dienes. Children to make the link of using tens as a replacement for the ones makes the answer ten times bigger as the representations has been unitised differently.



Unitising the ones in to tens showing that they are becoming ten times bigger.




















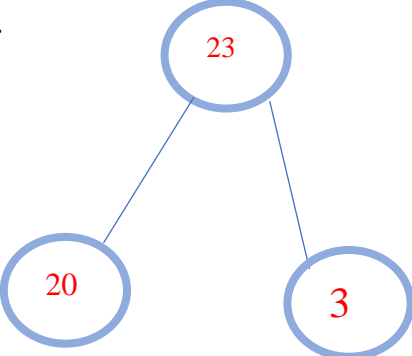













2 groups of 4 ones = 8 ones  
 2 groups of 4 tens = 8 tens

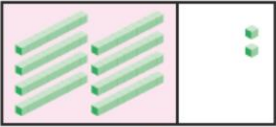
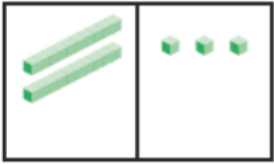
Children to use known number facts to support them when answering an equation where they are multiplying a one digit number by a multiple of ten.

$$4 \times 2 = 8$$

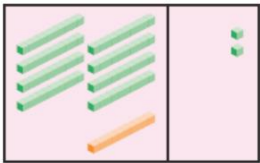
$$40 \times 2 = 80$$

	<p>So <math>2 \times 4 = 8</math>          And <math>2 \times 40 = 80</math></p>													
<p><b><u>Multiplication 2 digit number by a 1 digit number</u></b></p>  <table border="1" data-bbox="201 595 674 813"> <thead> <tr> <th></th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>		T	O										<p>Represent the place value grid pictorially.</p> <p>Partition the two digit number and then use known multiplication facts to multiply the two parts together and then add the products together.</p>  <p><math>20 \times 3 = 60</math></p> <p><math>3 \times 3 = 9</math></p> <p><math>60 + 9 = 69</math></p>	<p>Use expanded column method.</p> 
	T	O												
														
														
														
<p><b><u>Multiplying with regrouping</u></b></p> <p>Ensure that children understand that they will need to regroup the ones as there are more than 10 and therefore become 1 ten and the remaining ones.</p> <p><math>23 \times 4 =</math></p>		<p>use the expanded column method to support with renaming.</p> 												

3 x 4 = 12  
 12 = 1 ten and 2 ones



20 x 4 = 80



80 + 12 = 92

Multiply the ones

$$\begin{array}{r} \text{t} \quad \text{o} \\ 2 \quad 3 \\ \times \quad 4 \\ \hline 1 \quad 2 \\ 8 \quad 0 \end{array}$$

Multiply the tens

$$\begin{array}{r} \text{t} \quad \text{o} \\ 2 \quad 3 \\ \times \quad 4 \\ \hline 1 \quad 2 \\ + 8 \quad 0 \\ \hline 9 \quad 2 \end{array}$$

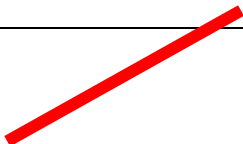
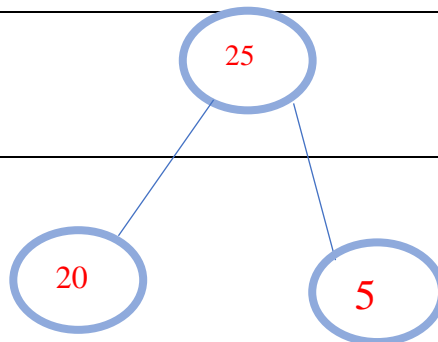
Add the ones and tens together to get the product.

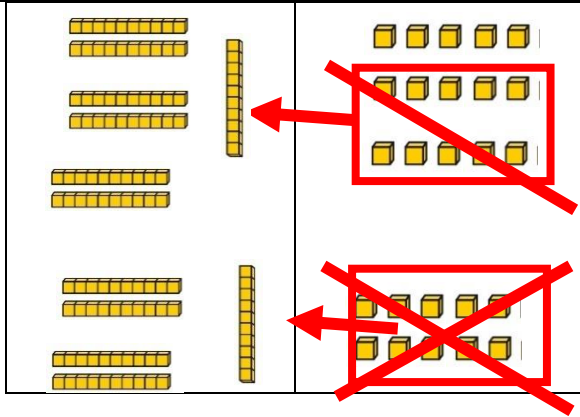
Conceptual Variation :

There are 25 green cubes in a box.  
 There are 5 times as many blue cubes than green cubes in the box.  
 How many cubes are there **altogether**?

T	O
---	---

h t o





$$20 \times 5 = 100$$

$$5 \times 5 = 25$$

$$100 + 25 = 125$$

$$\begin{array}{r} \times 25 \\ \underline{\phantom{0}5} \\ 25 \\ \underline{100} \\ 125 \end{array}$$



## Multiplication

**Key Language:** Double, times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as', array, commutative, factor

### Year 4

**National Curriculum**

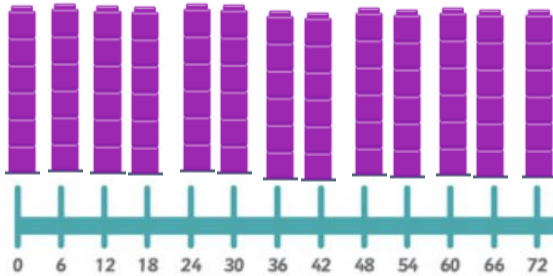
- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

**Resources**

- Counters
- Dienes
- Place value grids
- Number lines/tracks
- Multilink
- Part whole models
- Bar models
- Place value counters

### Concrete

**Counting in sixes, sevens and nines**



### Pictorial

When pupils know that the size of a group is 6, 7 and 9 and the group size remains consistent, they can count in multiples of 6, 7 and 9 to find the product.

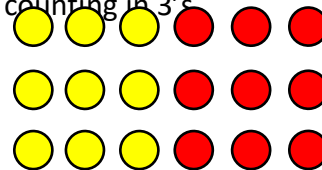
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

### Abstract

Count confidently without key manipulatives.

**Multiplying by 6**

Children to make the relationship from counting in 6's to counting in 3's



$3 \times 3 = 9$

Use recall to answer equations

$3 \times 6 = 18$

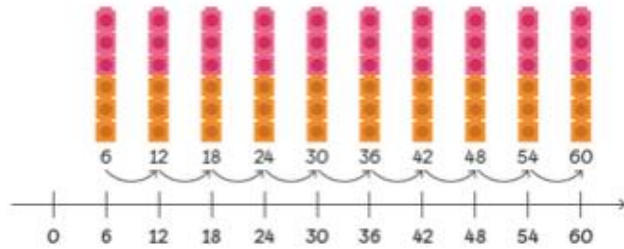
Children to support their quick recall by using known number facts

$3 \times 5 = 15$

$3 \times 6 = 15 + 3$

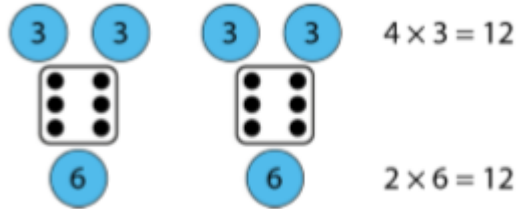
use number line to show that each time we add on an additional group we are adding on 6 to the

Count in sixes.



previous product.

$$3 \times 6 = 18$$



children to notice the relationship between x by 3 and x by 6. 6 is double 3.

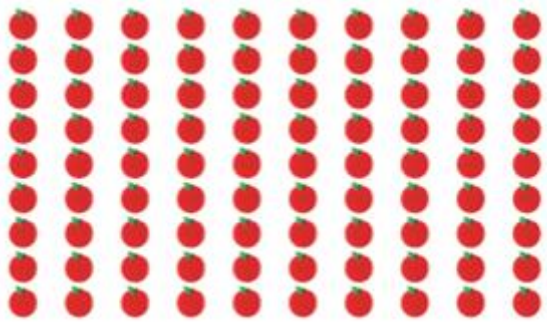
*'Six is double three, so \_\_\_ sixes are double \_\_\_ threes.'*

*'Three is half of six, so \_\_\_ threes are half of \_\_\_ sixes.'*

### Multiplying by 7

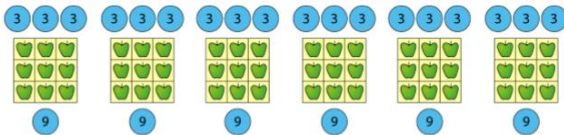
Repeat similar processes to multiplying by 6.

## Multiplying by 9

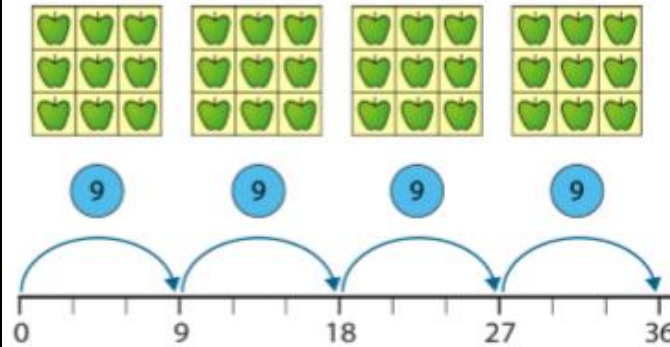


Children to create arrays using various counting objects.

Children to understand the 1 group of nine is the same as 3 groups of 3. Children can therefore use their 3 times tables to support them when counting in multiples of nines



Children to understand that when you are counting in groups of nine you are adding on nine to the previous factor.



Children to make the links between counting in threes through using bar models to support them.

3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
9			9			9			9			9			9		

Children to use quick recall to answer the equation.

Children can use know number facts to support them.

$$4 \times 9 = 45 - 9$$

$$1 \times 9 = 9$$

1 × 9 = 9  
↓  
×3

Use known facts within the three times tables to support answering equations in the nine times table.

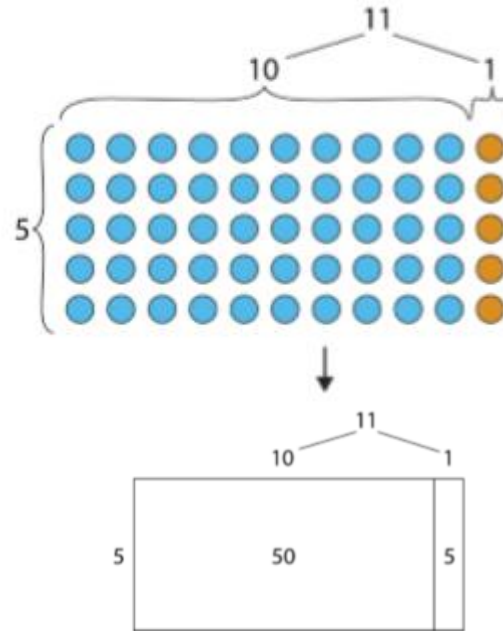
### Multiplying by 11

Children to use the distributive law to support them when counting in multiples of 11. Understand that

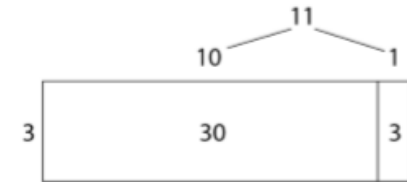


they are counting in 10's + 1.

use the distributive law through showing that they have partitioned 11 and multiplied each factor by 5 and then added them together.



Children to use bar models to support them in writing their equations and finding the product.



$$\begin{aligned} 11 \times 3 &= 10 \times 3 + 1 \times 3 \\ &= 30 + 3 \\ &= 33 \end{aligned}$$

Children to recall 11 times table facts using known facts to support them.

$$7 \times 11 = 55 + 11 + 11$$

### Multiplying by 12

Use methods similar to multiplying by 11

### Multiplying by 0 and 1

*'The product of \_\_\_ and zero is zero.'*

*'The product of zero and \_\_\_ is zero.'*

*The product of \_\_\_ and one is \_\_\_.'*

*The product of one and \_\_\_ is \_\_\_.'*

Children to understand that **when zero is a factor, the product is always zero.**

Represent pictorially

*'The product of \_\_\_ and zero is zero.'*

*'The product of zero and \_\_\_ is zero.'*

*The product of \_\_\_ and one is \_\_\_.'*

*The product of one and \_\_\_ is \_\_\_.'*

Children to understand that **when zero is a factor, the product is always zero.**

and  
**when one is a factor, the product is the same as the other factor.**



and  
**when one is a factor, the product is the same as the other factor.**

So

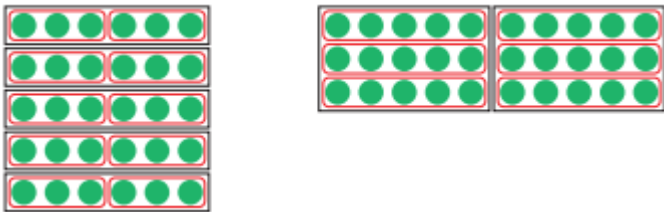
$5 \times 0 = 0$  because a factor is zero so the product will be zero

**And**

$5 \times 1 = 5$  as one of the factors is one therefore the remaining factor and the product are the same.

**Multiplying the same two numbers**

Arrays are used to support the understanding of commutativity. Pupils learn the pattern of  $a \times b = b \times a$ . Regardless of the order in which the factors are multiplied, the product remains the same.



Rotate the arrangement.



$3 \times 4$

$4 \times 3$

$3 \times 4 = 4 \times 3$

Children to abstract equations

$3 \times 4 = 12$  so  $4 \times 3 = 12$

I know  $3 \times 4 = 12$  so  $4 \times 3 = 12$

**multiplying 3 numbers**

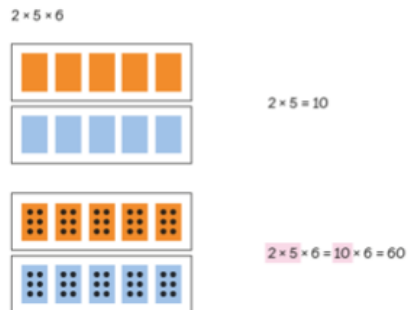
Children to use their understanding of commutativity to support them in solving each equation through finding the most efficient way to find the product.



Use knowledge of factors to simplify some multiplications.

$24 \times 5 = 12 \times 2 \times 5$

children to plot the array for to represent the first two factors within the equation and then represent the final factor within each part of the array.



$$2 \times 6 \times 10 = 120$$

$$12 \times 10 = 120$$

$$10 \times 6 \times 2 = 120$$

$$60 \times 2 = 120$$

Abstract the previous array to represent the equation.



$$12 \times 2 \times 5 =$$

$$\underbrace{\hspace{2cm}}_{12 \times 10} = 120$$

So,  $24 \times 5 = 120$

Children to continue to use their understanding of commutativity, known factors and use repeated addition if

Method 1  
 $3 \times 8 \times 5 =$  [ ]

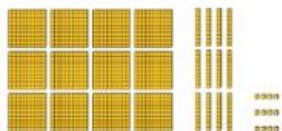
Method 2  
 $3 \times 8 \times 5 =$  [ ]



necessary.

**Multiplying by multiples of 10**

Children should have a good understanding for unitising numbers as 1's 10's or 100's



3 groups of 4 ones is 12 ones.  
 3 groups of 4 tens is 12 tens.  
 3 groups of 4 hundreds is 12 hundreds.

Represent pictorially.

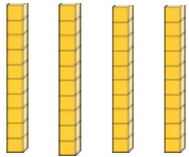
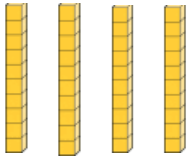
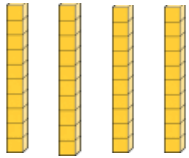
Children to use repeated addition understanding how many groups they are going to be adding and what will be in each group.

$$\begin{array}{r} 40 \\ 40 \\ \hline 40 \\ \hline 120 \end{array}$$

Use known number facts

Use dienes to represent/unitise the multiple of ten.  
Create groups to represent the equation.

$3 \times 40 =$



**Multiplying 2 digit numbers**

Partitioning

$10 \times 8 = 80$

10	3
10	3
10	3
10	3
10	3

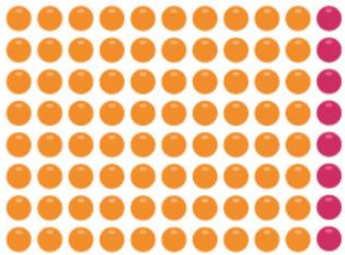
$3 \times 8 = 24$

Partitioning

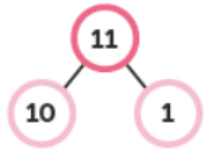
$3 \times 4 = 12$   
 $3 \times 40 = 120$

One factor is ten times bigger therefore the product will be 10 times bigger

**Partitioning- no renaming**

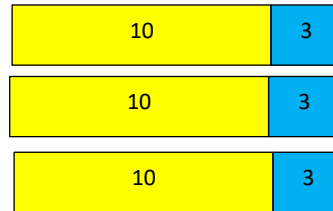


$8 \times 10 = 80$        $8 \times 1 = 8$



$8 \times 11 = 80 + 8$

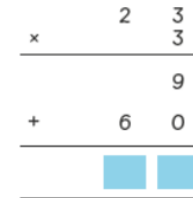
Children can partition 2 digit numbers into smaller factors to support them in using known facts to solve the equation.



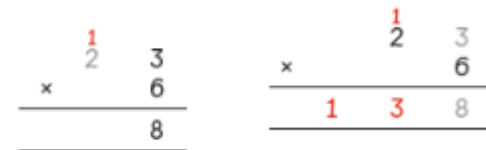
**Compact- renaming**

*Represent pictorially*

Children can use expanded column multiplication however they should be able to move on from this towards the end of the unit.



**Compact- renaming- children should be secure in this by the time they leave year 4**





Children can also use PV counters to support them in creating groups. Children can then mentally add the ones and then the tens.

**Compact- renaming**



use PV counters as you would have previously to support with using known

facts to multiply

Children to use their understanding of renaming to support them when using compact column multiplication.

**Multiplying by multiples of 100**

*Represent pictorially*

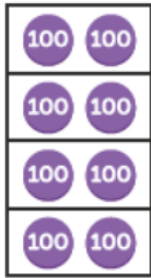
$$4 \times 2 = 8$$

$$4 \times 2 \text{ hundreds} = 8 \text{ hundreds}$$

$$4 \times 200 = 800$$

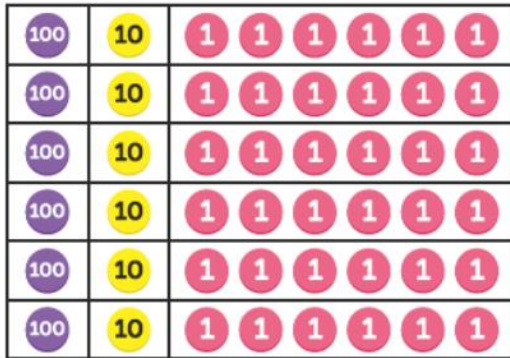
Children to use known facts to support them when multiplying by 100. The factor is 100 times more than the known fact therefore the product will be 100 times

$$4 \times 200 = 800$$



Children should be able to count in multiples of 100 and 2 and can use these to support them when using PV counters to multiply multiples of 100.

### Multiplying 3 digit numbers



$$100 \times 6 \quad 10 \times 6 \quad 6 \times 6$$

$$100 \times 6 = 600 + 60 + 36 = 696$$

$$10 \times 6 = 60$$

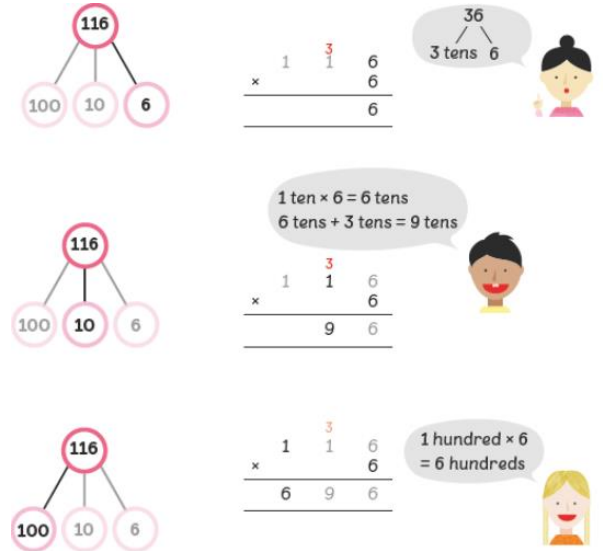
$$6 \times 6 = 36$$

$$\begin{array}{r} 116 \\ \times 6 \\ \hline 696 \end{array}$$

Represent pictorially

more than the product when multiplying with known facts.

use part whole models to support with unitising and therefore support when multiplying and renaming (supports most when moving away from expanded column



multiplication)

Children should be able to create the 3 digit number they are multiplying using PV counters and begin to create the amount of groups needed-looking at the other factor in the equation to know how many groups are needed.

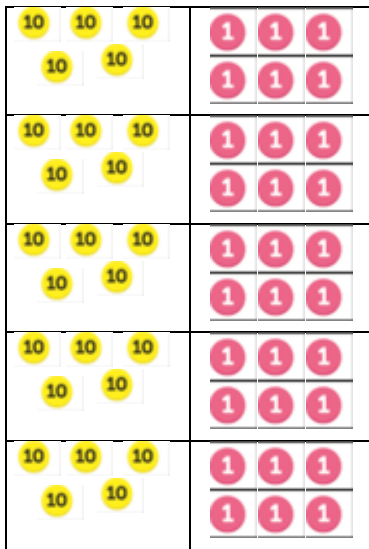
**Conceptual variation:**

Class 4 is selling tickets for a play.

Tickets cost £5 per person.

56 tickets have been sold so far.

How much money has Class 4 collected?



$$5 \times 50 = 250$$

$$5 \times 6 = 30$$

$$56 \times 5 = 280$$



$$\begin{array}{r} \phantom{0}^3 56 \\ \times 5 \\ \hline 280 \end{array}$$

£280		
------	--	--

**Multiplication**

**Key Language:**

Double, times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as', array, commutative, factor, square, cubed, prime, composite

**Year 5**

**National Curriculum**

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

**Resources**

- counters
- Dienes
- Place value grids
- Number lines/tracks
- Multilink
- Part whole models
- Bar models
- Place value counters
- Decimal place counters

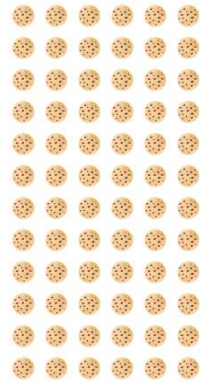
**Concrete**

**Pictorial**

**Abstract**

**Finding multiples**

Using counters and arrays to support understanding of multiples.



**Represent pictorially**

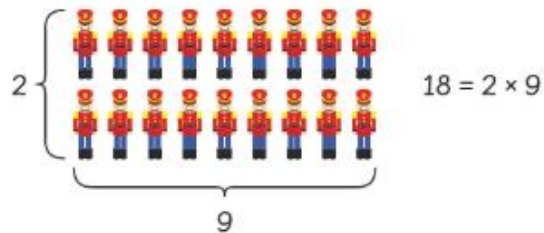
use understanding of times table facts to support understanding of multiples.

or

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72

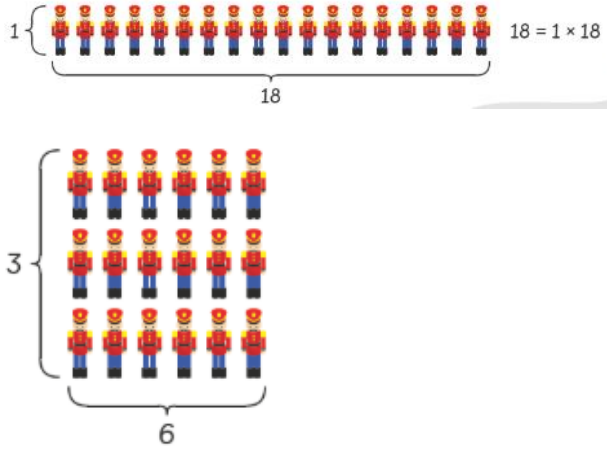
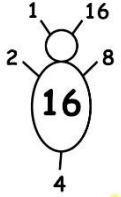

**Finding factors**

Spend time exploring how a product can be put into a variety of equal groups.



**Represent pictorially**

Use times table knowledge to support knowledge of factors.

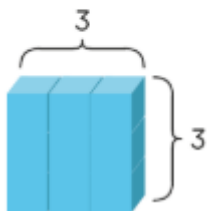
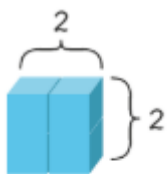
		
<p><b><u>Finding common factors</u></b></p> <p>Finding factors using methods above</p>	<p><i><u>Represent pictorially</u></i></p>	
<p><b><u>Finding prime numbers/composite numbers</u></b></p> <p>Spent time exploring how products can be put into equal groups- drawing attention to those who only have two factors.</p>  <p><math>5 = 1 \times 5</math></p>	<p><i><u>Representing pictorially</u></i></p>	<p>Use known multiplication facts to support with prime numbers</p>



$$7 = 1 \times 7$$

Composite numbers are numbers which are **not** prime.

**Finding square and cubed numbers**  
using cubes or counters to support understanding of cubed numbers



*Represent pictorially*

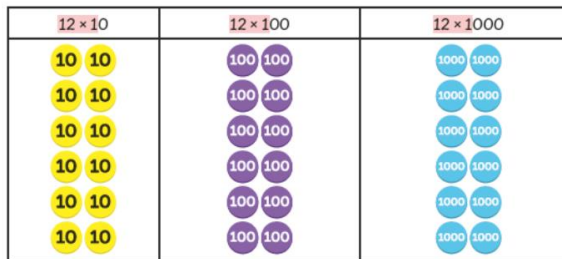
identify patterns using a multiplication grid.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

**Multiplying by 10, 100 and 1000**

*Represent pictorially*

Children to use their understanding from unitising with PV equipment and their understanding of known facts to support them when multiplying by 10, 100 or 1000



spend time unitising using PV equipment to support. Dienes blocks are good here as they show each unit as ten times bigger than the unit before.

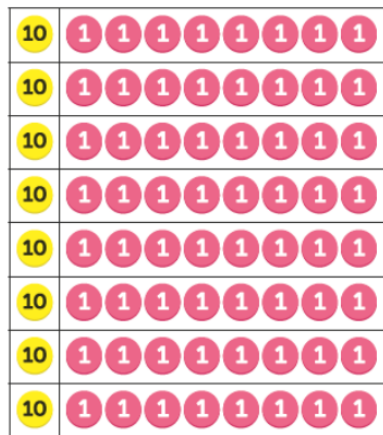
$$12 \times 10 = 120$$

$$12 \times 100 = 1200$$

$$12 \times 1000 = 12000$$

**Multiplying by 2 digit and 3 digit numbers by a single digit**

*Represent pictorially*



$18 \times 8 =$



$118 \times 3 =$

$$\begin{array}{r}
 118 \\
 \times 3 \\
 \hline
 24 \rightarrow \text{multiply by ones} \\
 30 \rightarrow \text{multiply by tens} \\
 + 300 \rightarrow \text{multiply by hundreds} \\
 \hline
 354
 \end{array}$$

$$\begin{array}{r}
 18 \\
 \times 8 \\
 \hline
 64 \rightarrow \text{multiply by ones} \\
 80 \rightarrow \text{multiply by tens} \\
 \hline
 144
 \end{array}$$

Or (this should be secure by the time children leave)

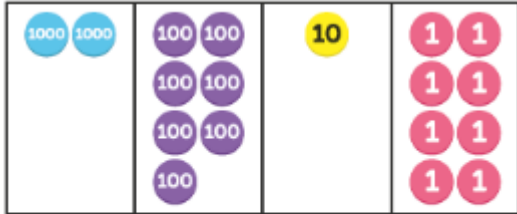


$$\begin{array}{r} 118 \\ \times 3 \\ \hline 354 \end{array}$$

$$\begin{array}{r} 18 \\ \times 8 \\ \hline 144 \end{array}$$

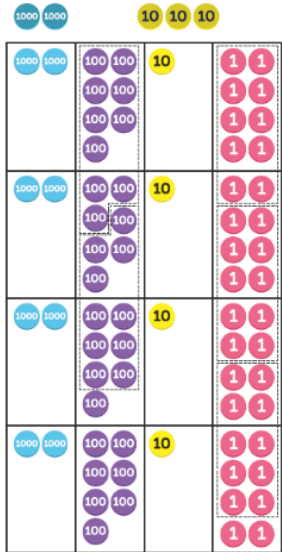
**Multiplying 4 digit numbers**

2718 x 4 =



2 thousands    7 hundreds    1 ten    8 ones  
 ↓                    ↓                    ↓                    ↓  
 8 thousands   28 hundreds   4 tens    32 ones

or



*Represent pictorially*

$$\begin{array}{r}
 2718 \\
 \times \quad 4 \\
 \hline
 32 \\
 40 \\
 2800 \\
 + 8000 \\
 \hline
 10872
 \end{array}$$

n to use their

$$\begin{array}{r}
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}\phantom{7}\phantom{1}2 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}\phantom{7}72 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}872 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}10872
 \end{array}$$

$$\begin{array}{r}
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}\phantom{7}\phantom{1}2 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}\phantom{7}72 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}872 \\
 \hline
 \phantom{2}7\overset{3}{1}8 \\
 \times \phantom{2}\phantom{7}\phantom{1}4 \\
 \hline
 \phantom{2}10872
 \end{array}$$

**Multiplying a 2 digit number by a 2 digit number**

use of place value counters to support children partitioning one of the two digit numbers and then using known facts to multiply.

$14 \times 12 =$



$14 \times 10$   
 $= 14 \times 1 \text{ ten}$   
 $= 14 \text{ tens}$



$14 \times 12 = 168$



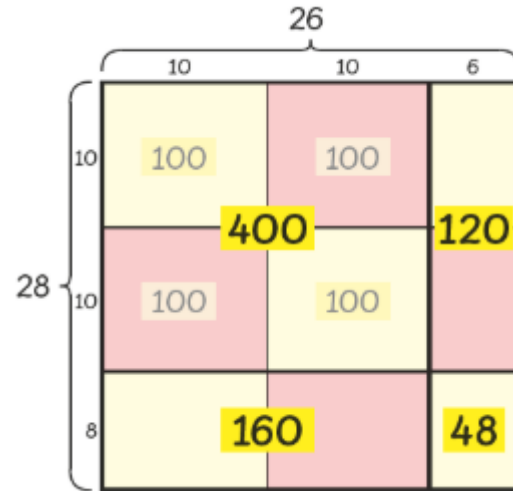
$14 \times 2 = 28$

$14 \times 10 = 140$

$14 \times 2 = 28$

$14 \times 12 = 168$

'grid method' approach with children.



understanding of unitising to support them when they are using short multiplication.

$$\begin{array}{r} 4 \\ 28 \\ \times 26 \\ \hline 8 \end{array} \rightarrow \begin{array}{r} 4 \\ 28 \\ \times 26 \\ \hline 168 \end{array}$$

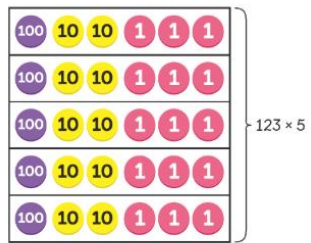
$$\begin{array}{r} 1 \\ 28 \\ \times 26 \\ \hline 168 \\ 6 \end{array} \rightarrow \begin{array}{r} 1 \\ 28 \\ \times 26 \\ \hline 168 \\ 56 \end{array}$$

$$\begin{array}{r} 1 \\ 4 \\ 28 \\ \times 26 \\ \hline 168 \\ + 56 \\ \hline 728 \end{array}$$

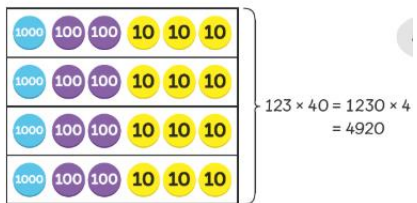
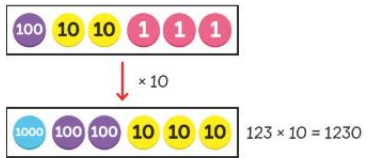
$\rightarrow 28 \times 6$   
 $\rightarrow 28 \times 20$

### Multiplying a 3 digit number by a 2 digit number

$$\begin{array}{r} 123 \\ \times 45 \\ \hline \end{array}$$



$$123 \times 5 = 500 + 100 + 15 = 615$$



### Represent pictorially

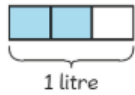
$$\begin{array}{r} 123 \\ \times 45 \\ \hline 615 \quad \rightarrow \text{multiply by 5} \\ + 4920 \quad \rightarrow \text{multiply by 40} \\ \hline 5535 \end{array}$$

$$123 \times 45 = 5535$$

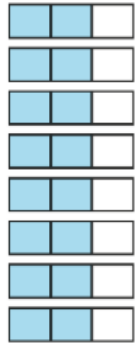
with renaming

$$\begin{array}{r} 11 \\ 245 \\ \times 13 \\ \hline 735 \quad \rightarrow \text{multiply by 3} \\ + 2450 \quad \rightarrow \text{multiply by 10} \\ \hline 3185 \end{array}$$

**Multiplying fractions by whole numbers**



use bar models to support multiplication of fractions by a whole number



**Represent pictorially**

multiplying the numerator by the whole number.

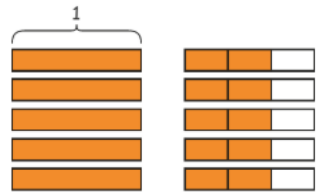
$$\begin{aligned} 8 \times \frac{2}{3} &= 8 \times 2 \text{ thirds} \\ &= 16 \text{ thirds} \\ &= \frac{16}{3} \end{aligned}$$

$$\frac{16}{3} = 5 \frac{1}{3}$$

**Multiplying mixed numbers**

Children to draw bar models to support their understanding.

$$1 \frac{2}{3} \times 5 =$$



  $5 \times 1 = 5$

  $5 \times 2 \text{ thirds} = 10 \text{ thirds}$

*Represent pictorially*

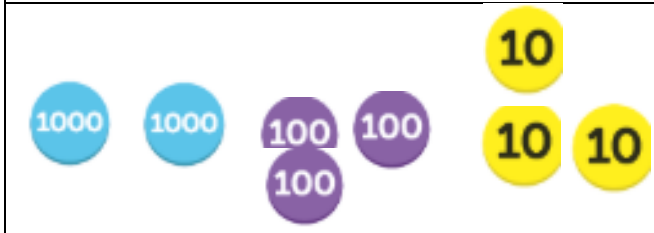
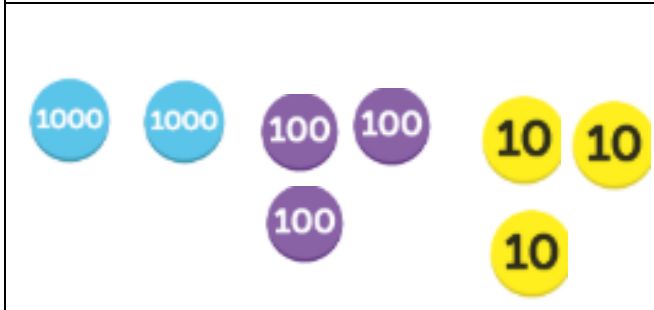
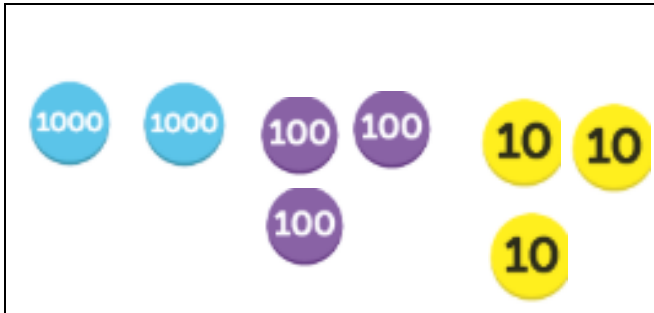
children to multiply the numerator by the whole number and then the mixed number by thre whoole number- simlify the fraction if needed.

$$\begin{aligned} 5 \times 1 \frac{2}{3} &= 5 + \frac{10}{3} \\ &= 5 + 3 \frac{1}{3} \\ &= 8 \frac{1}{3} \end{aligned}$$

**Conceptual variation 234 x 44**

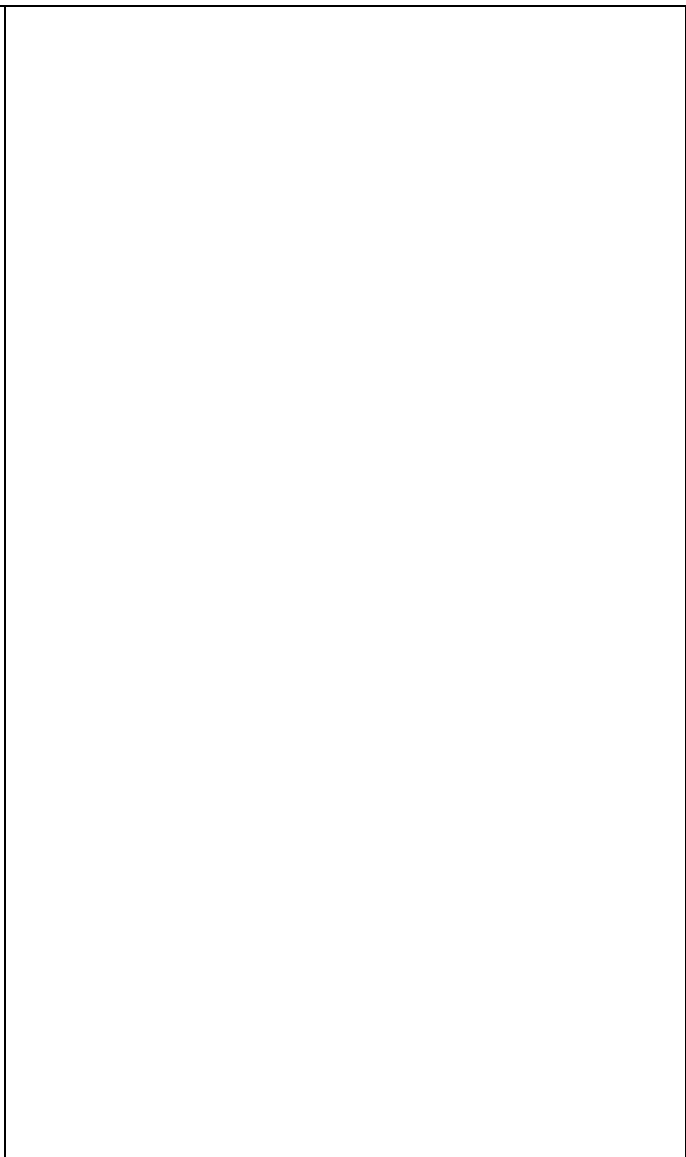
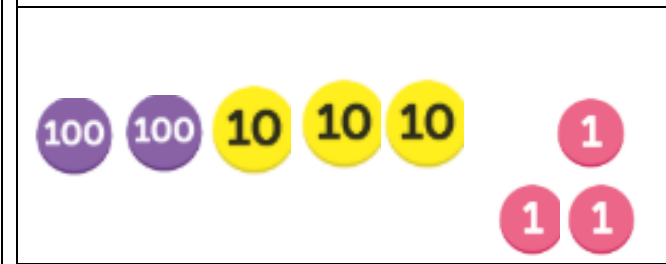
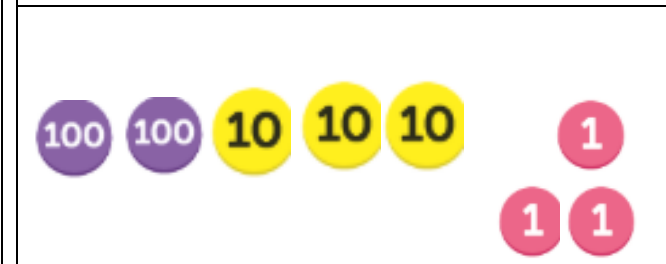
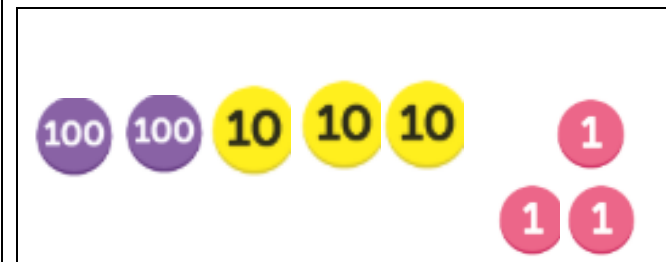
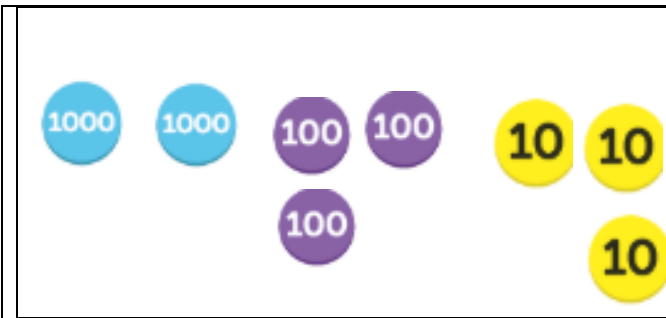


**X 10**



Show pictorially

$$\begin{array}{r}
 \overset{1}{2} \overset{1}{3} \overset{1}{4} \\
 \times 44 \\
 \hline
 936 \text{ (} 234 \times 4 \text{)} \\
 \underline{9360} \text{ (} 234 \times 40 \text{)} \\
 10296
 \end{array}$$





Show renaming within each column

## Multiplication

**Key Language:** Double, times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as', array, commutative, factor, square, cubed, composite

### Year 6

#### National Curriculum

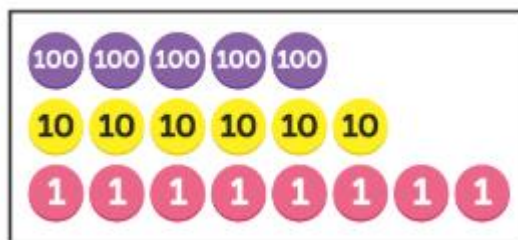
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,  $4 \frac{1}{2} \times 2 \frac{1}{2} = 8 \frac{1}{2}$ ]
- multiply one-digit numbers with up to two decimal places by whole numbers

#### Resources

- counters
- Dienes
- Place value grids
- Number lines/tracks
- Multilink
- Part whole models
- Bar models
- Place value counters
- Decimal place counters

#### Concrete

#### Multiplying 2/3/4 digit numbers by a 2 digit number



$$12 \times 568 =$$

Children to spend time unitising- recalling 10 ones= 1 ten etc

Children to use place value counters to support them when multiplying

#### Pictorial

*Represent pictorially*

#### Abstract

$$\begin{array}{r}
 568 \\
 12 \\
 \hline
 16 \\
 120 \\
 1000 \\
 80 \\
 600 \\
 \hline
 5000
 \end{array}$$

Or compact column multiplication

$$\begin{array}{r}
 568 \\
 \times 12 \\
 \hline
 1136 \rightarrow 568 \times 2 \\
 + 5680 \rightarrow 568 \times 10 \\
 \hline
 6816
 \end{array}$$

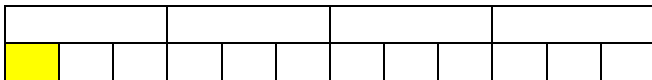
(Step by step)

$24 \times 2568 =$

$$\begin{array}{r}
 2568 \\
 \times 24 \\
 \hline
 10272 \\
 + 51360 \\
 \hline
 61632
 \end{array}$$

**Multiplying fractions**

$\frac{1}{4} \times \frac{1}{3}$



Divide each of the quarters in to 1/3 as all parts must be equal. Look at the section of the whole needed to find a 1/3 of and shade a 1/3 of each of the parts of the whole.

*Represent pictorially*

$\frac{1}{2} \times \frac{1}{3} =$

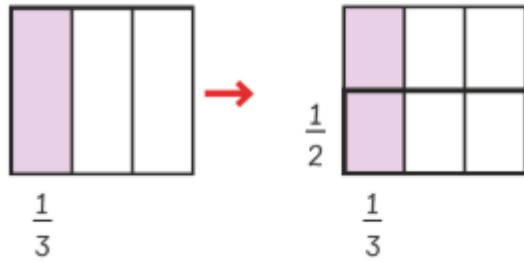
Multiply the numerators then the denominators.

$1 \times 1 = 1$   
 $2 \times 3 = 6$

Simplify if possible.

$\frac{1}{3}$  of a  $\frac{1}{2}$   
 $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Children to create models to support them when x a



fraction.

**Multiplying decimals**



*Represent pictorially.*

**Method 1**

$$0.2 \times 3 = 0.6$$

$$0.03 \times 3 = 0.09$$

---


$$0.23 \times 3 = 0.6 + 0.09 = 0.69$$

**Method 2**

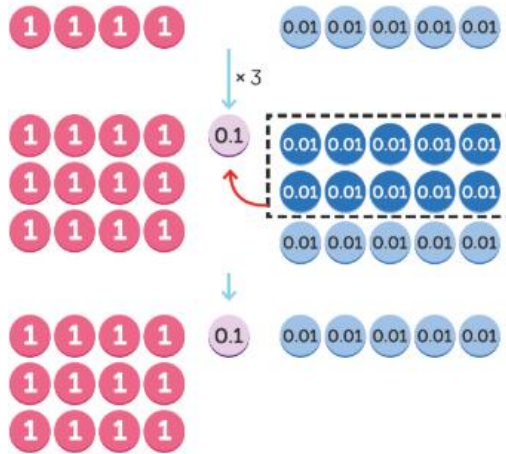
$$0.23 = 23 \text{ hundredths}$$

$$0.23 \times 3 = 23 \text{ hundredths} \times 3$$

$$= 69 \text{ hundredths}$$

$$= 0.69$$

**With renaming**



**With renaming**

$$\begin{array}{r} 4.05 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4.05 \\ \times \quad 3 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 4.05 \\ \times \quad 3 \\ \hline 12.15 \end{array}$$

**Multiplying decimals by a 2 digit number**

$$3.4 \times 16 =$$

$$3.4 \times 10 = 34$$

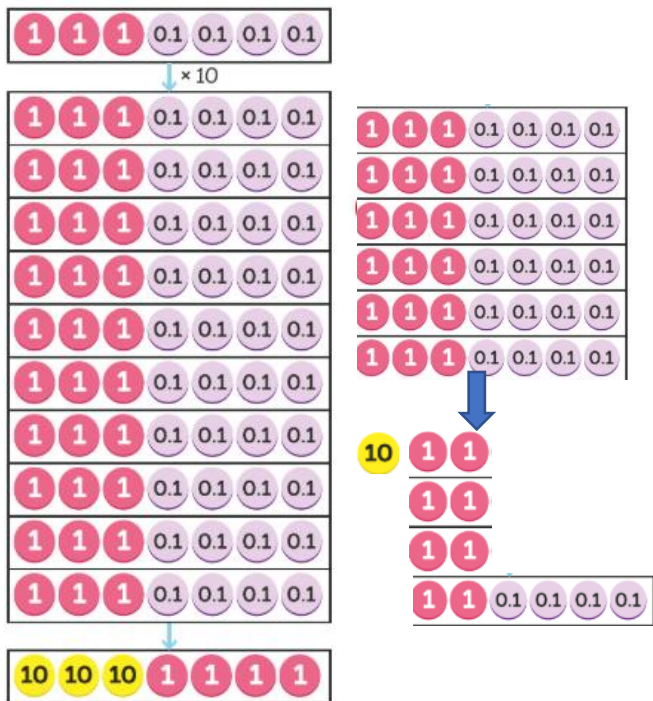
*Represent pictorially*

$$\begin{array}{r} 3.4 \\ \times \quad 6 \\ \hline 2.4 \\ + 18 \\ \hline 20.4 \end{array}$$

Or

$$\begin{array}{r} 2 \\ 3.4 \end{array}$$

$$3.4 \times 6 = 20.4$$






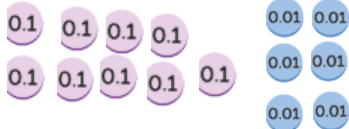
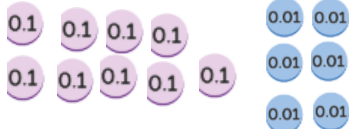

$$\begin{array}{r} 6 \\ \hline 24.4 \\ \hline \end{array}$$

**Conceptual variation:  $1.96 \times 7$**

Represent pictorially



$$\begin{array}{r} \overset{6}{1}.\overset{4}{9}6 \\ \times 7 \\ \hline 13.72 \\ \hline \end{array}$$

<p>1</p> 		
<p>1</p> 		
<p>1</p> 		
<p>1</p> 		
<p>1</p> 		
<p>1</p> 		

Show renaming in each column

$$0.06 \times 7 = 0.42$$

$$0.9 \times 7 = 6.3$$

$$1 \times 7 = 7$$

$$0.42 + 6.3 + 7 = 13.72$$